

Mathematics, Science, and Reality

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Intention for this Presentation



Taking Care of the Details

- In mathematics we spend a lot of time covering the details.
- After all, putting the decimal point in the “correct place” can make a big difference!



Teaching a Rich Curriculum

- As a result, most of our time is spent studying the trees.
- Less time is spent studying the forest.

The View Above the Trees

- This presentation “looks at the forest”, rather than the trees.



Communicating the Big Picture

- ❑ Eliminates the *disconnect* people have with math
- ❑ Guides us as we *make decisions for students*



What is Reality?

Concepts about Reality

- ❖ Many times in society,
“Reality is *ala carte*.”
- ❖ Many times in religion,
“Reality is *absolute*.”
- ❖ Very often,
“Reality is *a lot of agreement*.”



Reality in this Presentation

- ❖ Concentrates on the *physical reality* as described by math and science
- ❖ A math/science model should make a *prediction* and be *falsifiable*.



Science and Reality

- ❖ Science depends on empirical evidence.
- ❖ Observations typically come before equations.
In this sense, math is a “tool” of science.



Reality Determined by the Senses

- ❖ Our senses have help us to determine many aspects of reality — reality can be intuitive.
- ❖ For example, baseball is intuitive.



Reality Determined by Technology and Math

- ❖ There is no reason to believe that reality continues to be intuitive when we use technology or math.
- ❖ For example, black holes are nonintuitive.

Transcendental Numbers



← Nonintuitive

Mathematics Representing Reality

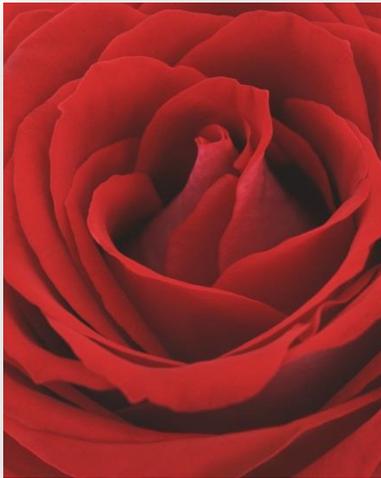


Mathematics and Color

- Color might appear “at first glance” to be outside the realm of mathematics.
- However, mathematics describes color by using *ordered triples*.

Representing Color

- White light is **red**, **green**, and **blue** light. Monitors create at least 256 different intensities of these colors, numbered 0 to 255. Colors are given by (r, g, b) .



$(255, 0, 0) = \underline{\text{Red}}$

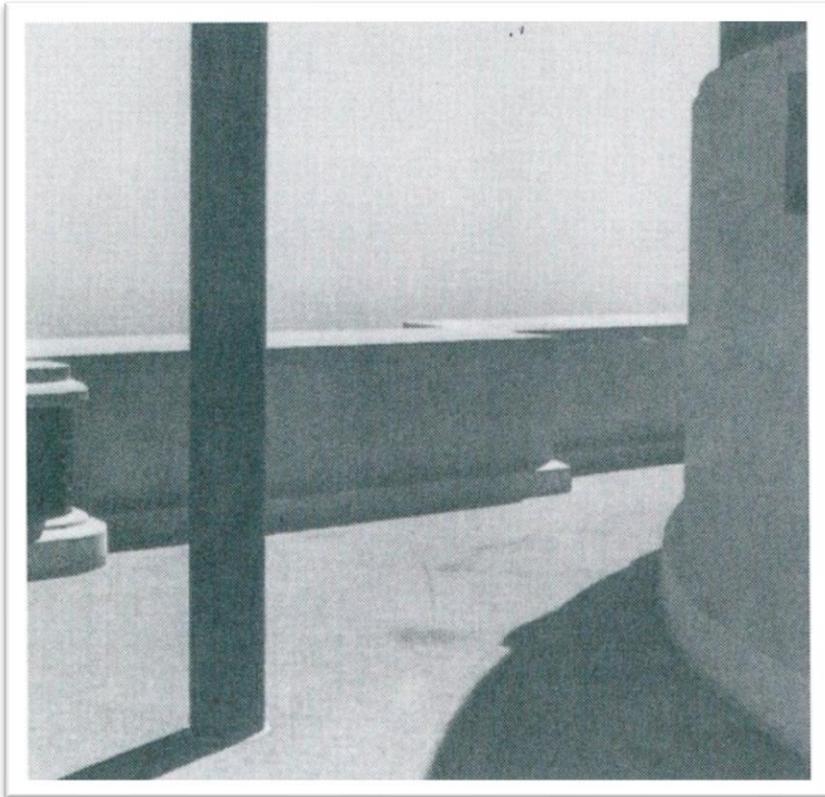


$(255, 255, 0) = \underline{\text{Yellow}}$

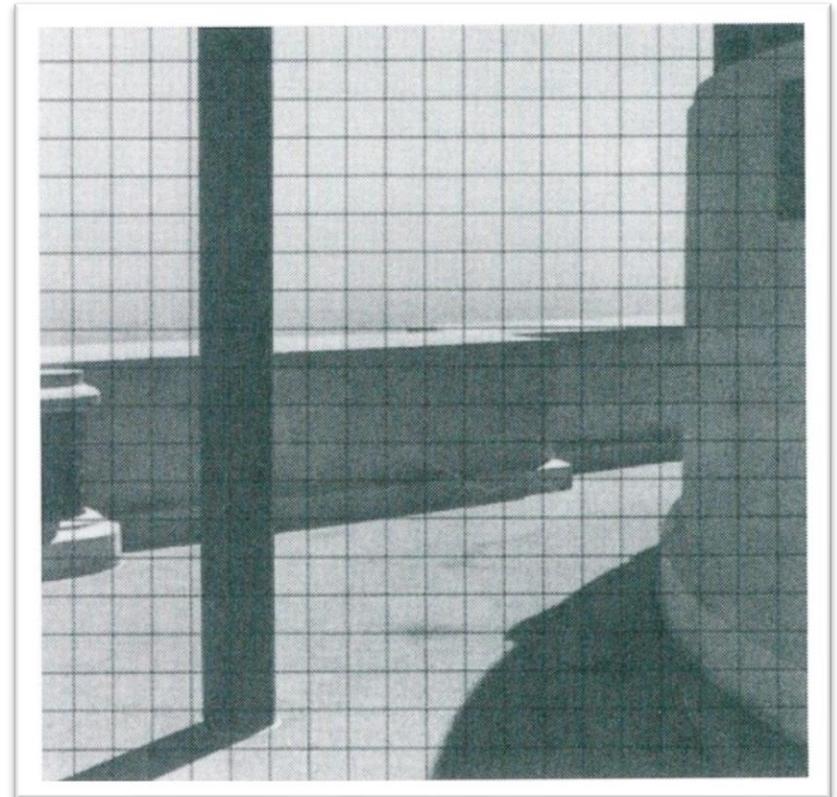
- $256 \times 256 \times 256 = 16,777,216$ different colors (TrueColor)

Analog Versus Digital

Given B/W Analog Photo



Overlay a grid.



Converting to Digital

- ❖ Establish a gray scale.
- ❖ Determine the scale number for each square.

Digitized Photo



1	1	1	1	10	11	1	1	1	1	1	1	1	1	1	1	1	1	9	8	
1	1	1	1	10	11	1	1	1	1	1	1	1	1	1	1	1	1	2	8	9
1	1	1	1	10	11	1	1	1	1	1	1	1	1	1	1	1	1	4	8	10
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3	5	9	8	10	11	7	7	7	7	8	8	8	7	6	6	6	6	8	8	
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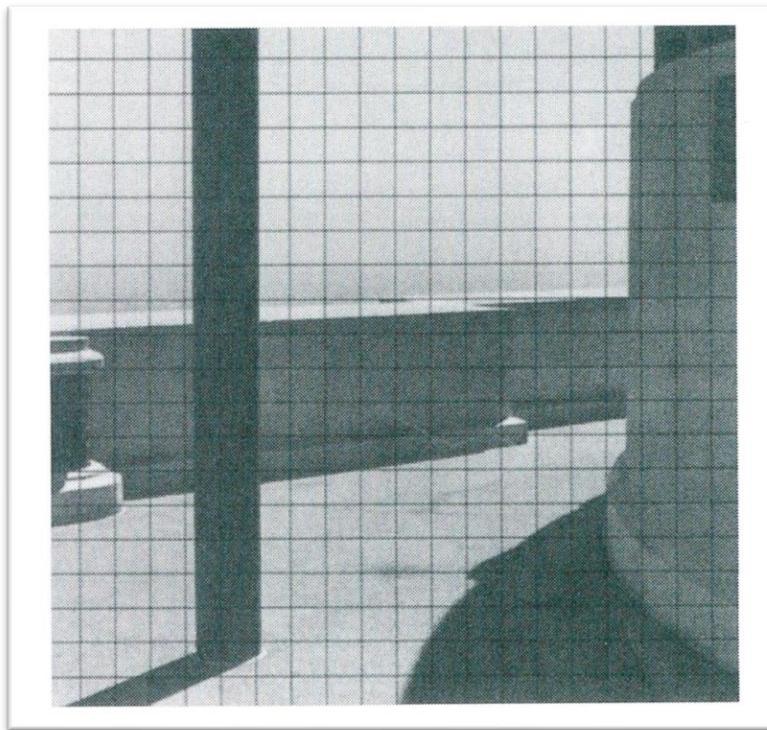
Gray Scale



0	1	2	3	4	5	6	7	8	9	10	11
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Converting Analog to Digital

Analog Photo



Digital Photo

1	1	1	1	10	11	1	1	1	1	1	1	1	1	1	1	1	1	9	8	
1	1	1	1	10	11	1	1	1	1	1	1	1	1	1	1	1	1	2	8	9
1	1	1	1	10	11	1	1	1	1	1	1	1	1	1	1	1	1	4	8	10
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2	2	5	7	9	8	2	2	2	2	7	9	9	9	10	11	11	11	10	7	
7	9	7	5	2	2	2	2	2	2	8	9	9	10	11	11	11	11	11	10	



How Much Does a Human “See” in a Lifetime?

- ❖ Movies show 48 frames per second.
- ❖ During 100 years a person might see the equivalent of 150 billion frames.

How Many Pixels Does a Human “See” in a Lifetime?

- ❖ Eye resolution is 576 megapixels per frame.
- ❖ In a lifetime, 150 billion frames represents **87 trillion pixels.**



But Sight Requires Interpretation!?

- Consider self driving cars



Making Sense Out of the Pixels

- Cameras bringing data at 30 fps that must be analyzed in *REAL TIME*.



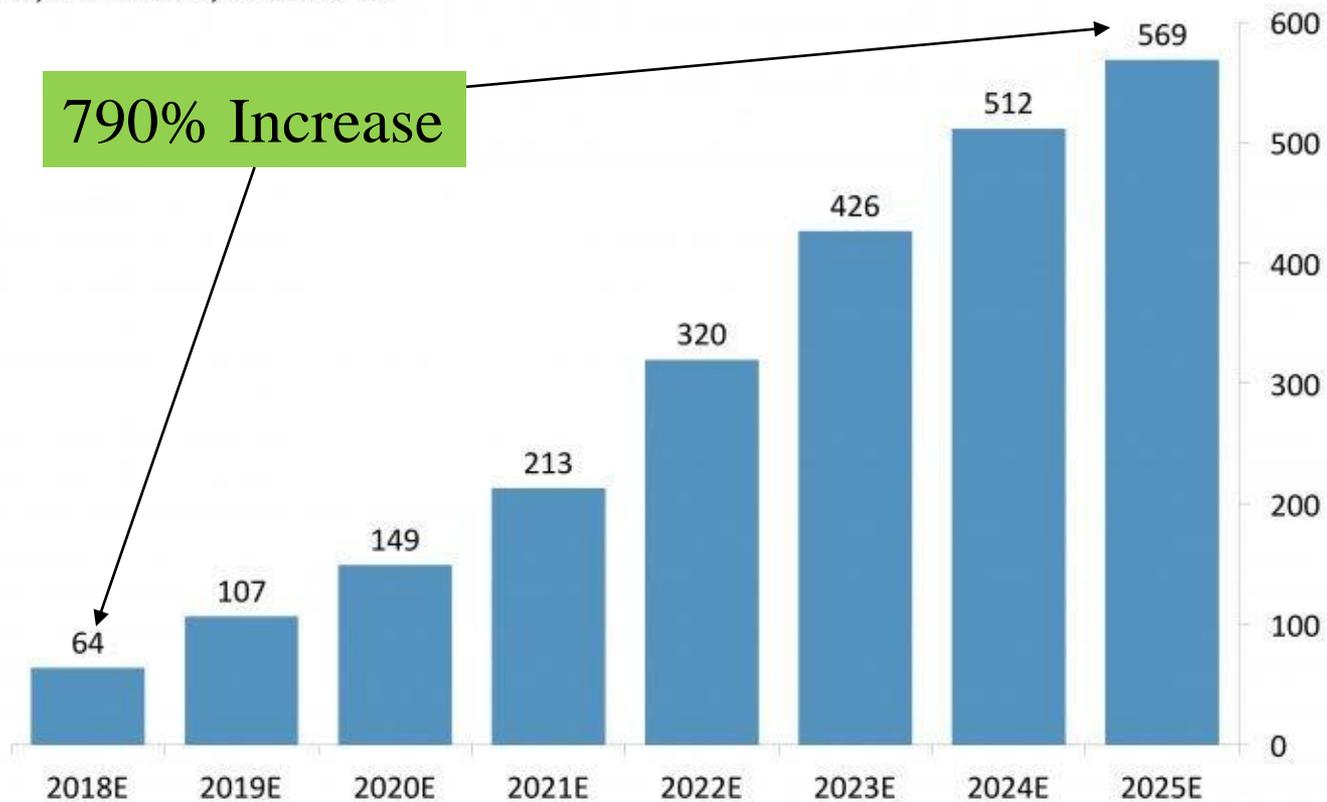
Advanced Math and Computation

- Small supercomputers are now available.
- Math algorithms are essential for this technology.

Future of Autonomous Cars

FORECAST: Fully Autonomous Car Shipments

US, 2018-2025, Thousands



Source: BI Intelligence Estimates, 2017

BI INTELLIGENCE



Artificial Intelligence Is Key

- “Tech Giants Are Paying Huge Salaries for Scarce A. I. Talent” --*New York Times*, October 22, 2017
- Some salaries for A. I. specialists: \$300-500K

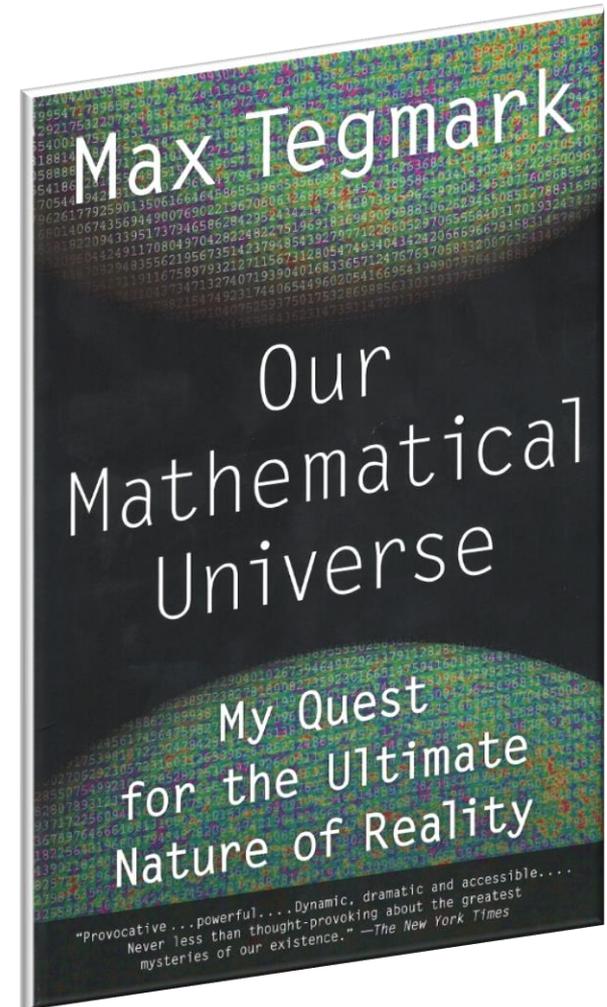


Virtual Reality

- Are self driving cars a case where virtual reality tries to interpret physical reality?

Sight: A Mathematical Structure?

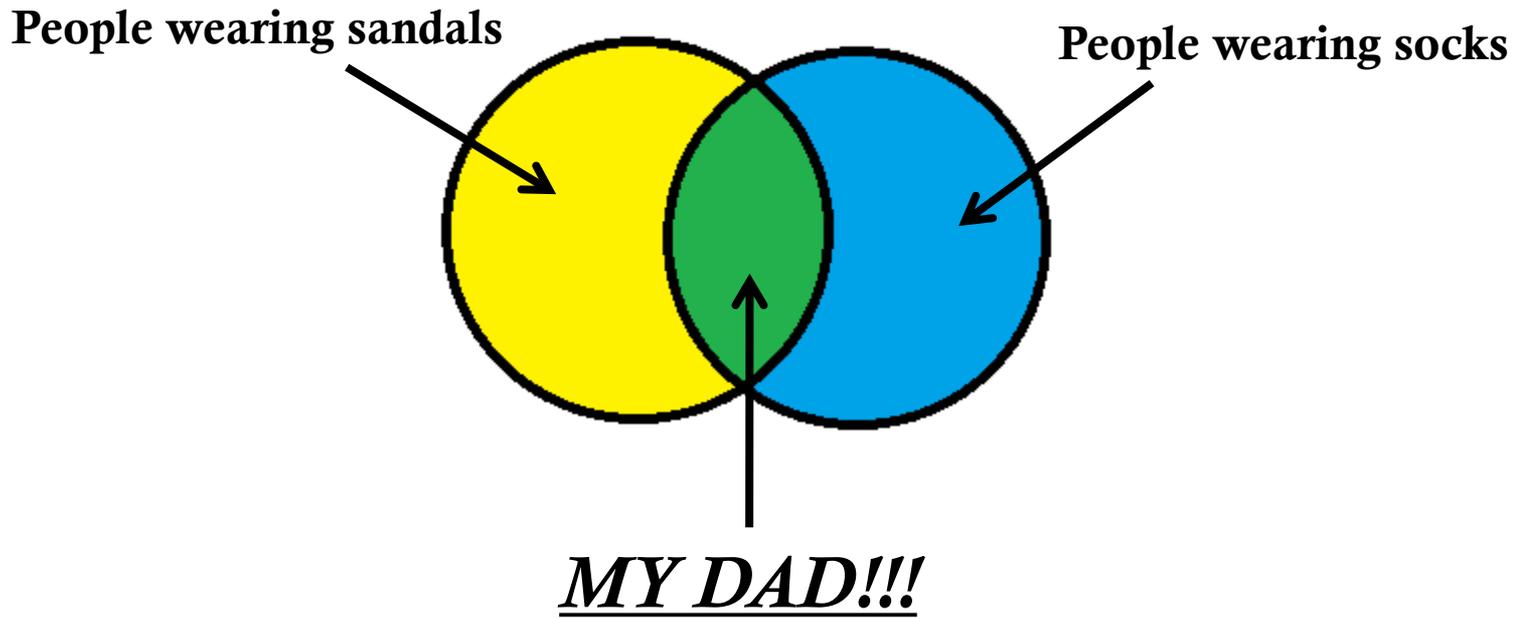
- If mathematics can *accurately describe* a physical phenomenon, then is this mathematical structure *isomorphic* to a portion of physical reality?



**Human Behavior
and
Mathematical Structures**

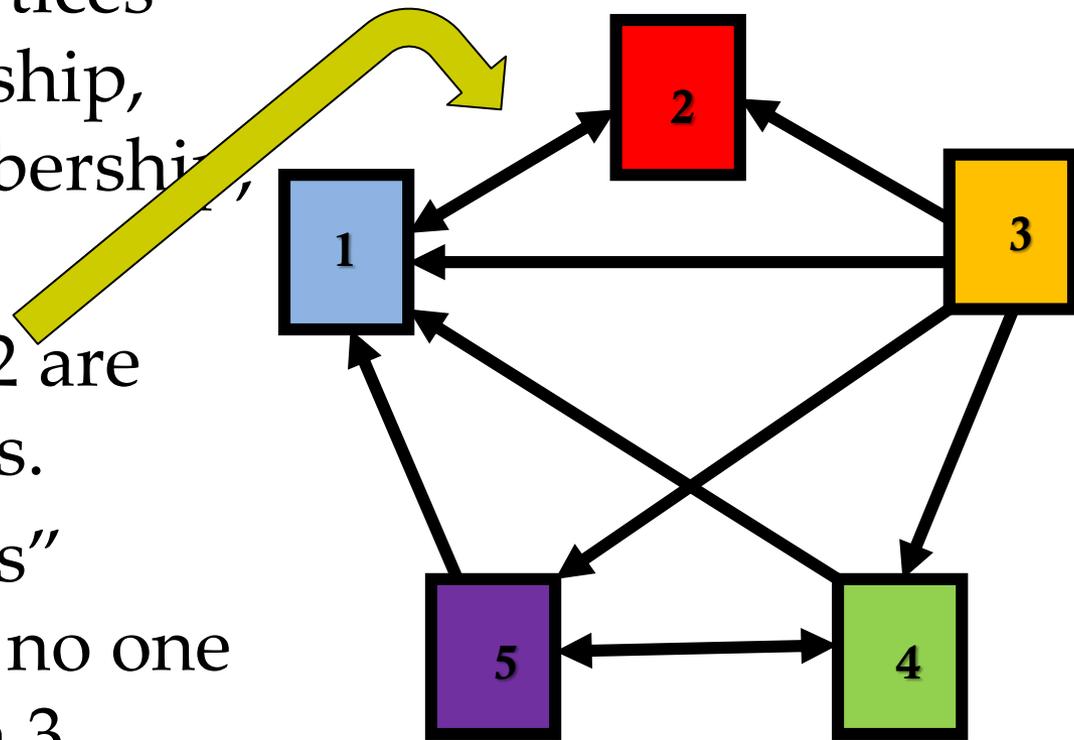
Modeling Social Situations

Consider teaching the concept of intersection in a Venn diagram in today's classroom.



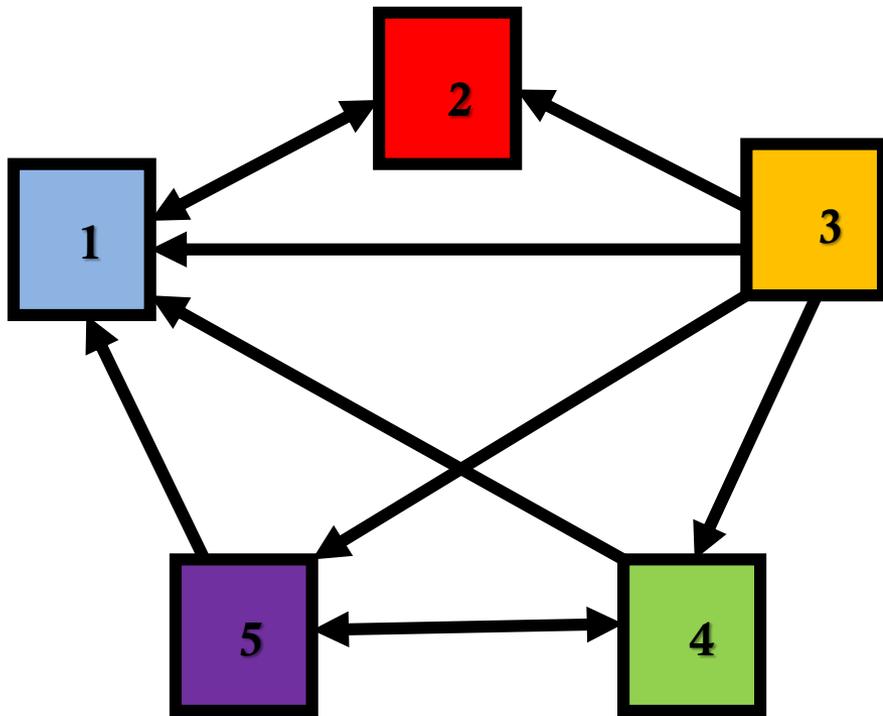
Social Networks as Directed Graphs

- ❖ People are vertices tied by friendship, interest, membership, or beliefs.
- ❖ Person 1 and 2 are mutual friends.
- ❖ Person 3 “likes” everyone, but no one “likes” Person 3.



Adjacency Matrices

- ❖ A *square* adjacency matrix can be used to summarize this simple social network.

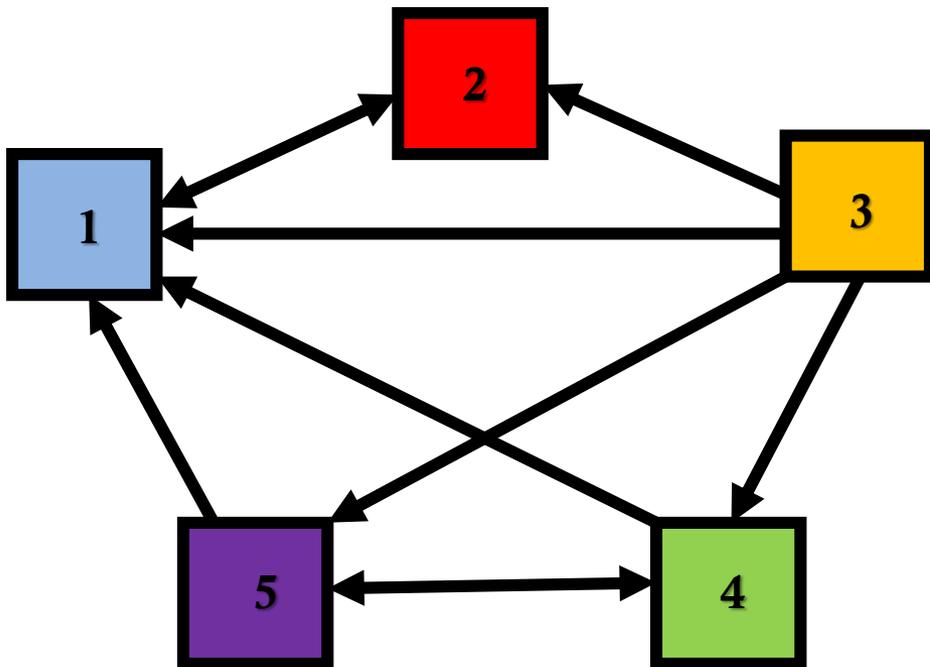


$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Calculating Social Distance

The computation A^2 can be used to determine people who have a social distance of 2.

Person 3 has a social distance of 2 to Person 1, three different ways.



$$A^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

The Power of Mathematics

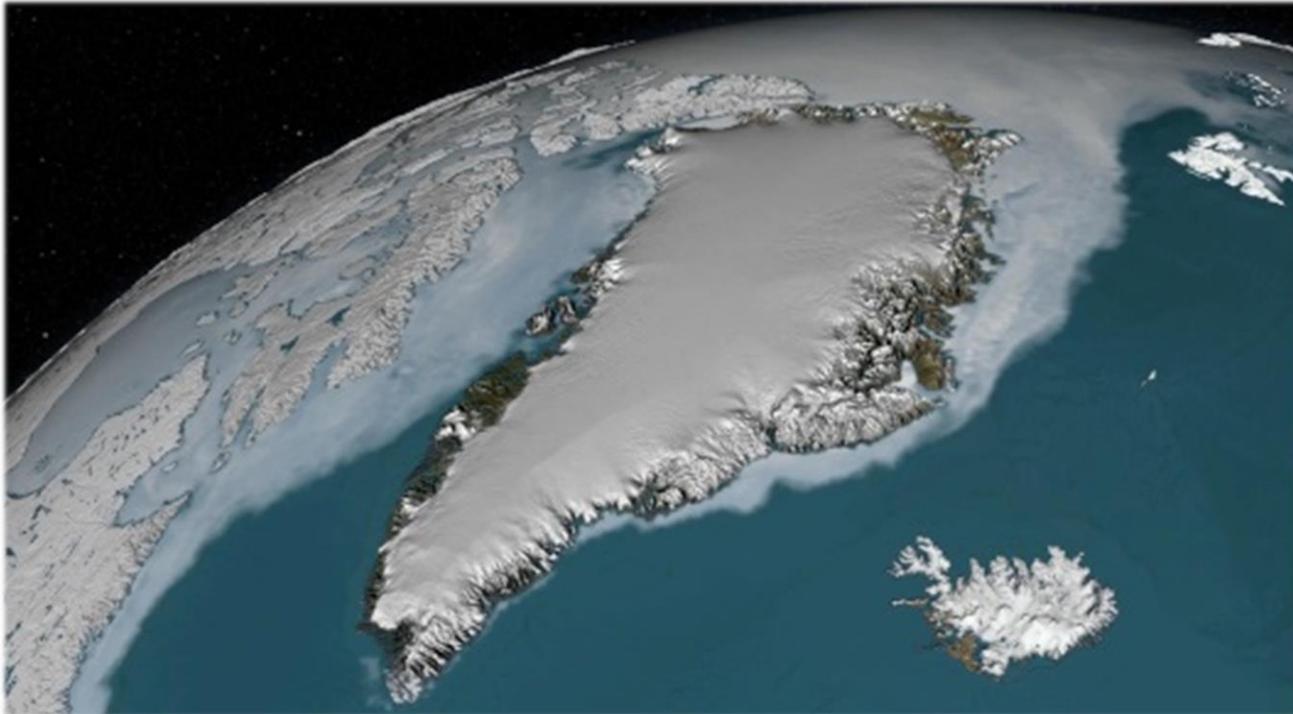


Sometime We Forget that ...

- ❖ ... the Empire State Building was made largely with slide rule technology, ...

Rise in Sea Level

- ❖ We can calculate the rise in sea level using basic geometry and algebra.





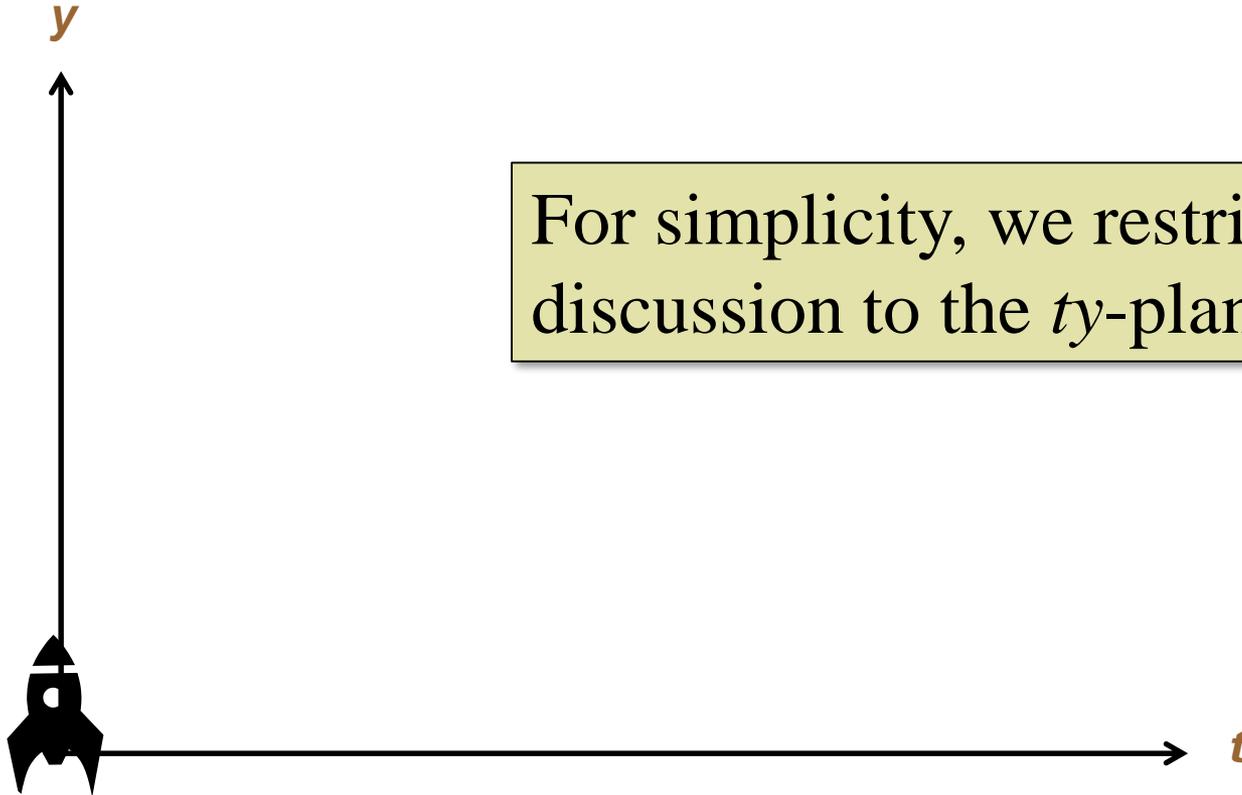
The Power of Mathematics

- ❖ Einstein derived special relativity using math that was not more than *freshman calculus*.

Finding Nonintuitive Reality

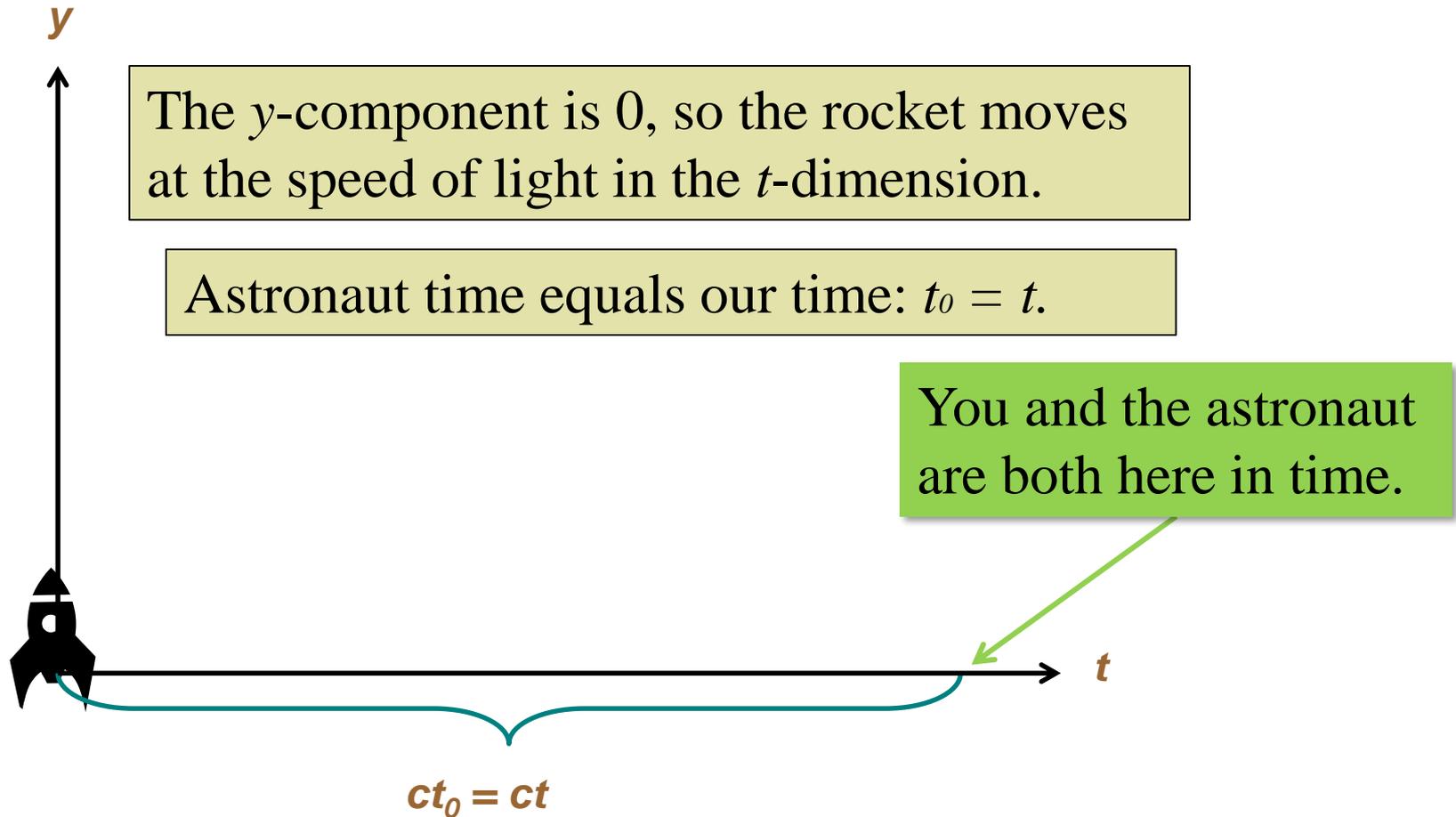
- Einstein's special relativity assumptions:
 - *The speed of light c is constant.*
 - *We are all moving at the speed of light in four dimensional space-time.*

A Rocket Located in Time and One-Dimensional Space

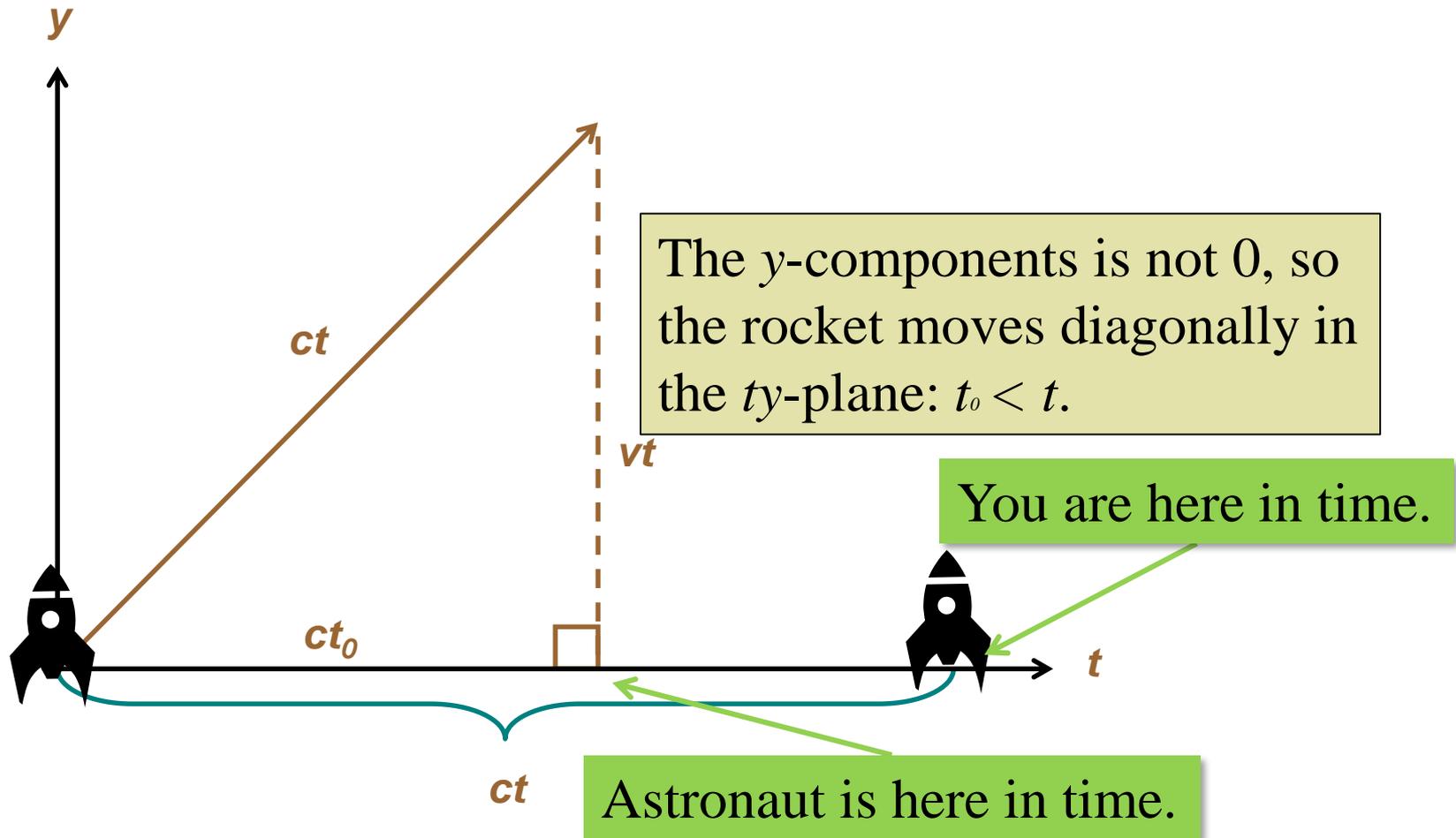


For simplicity, we restrict our discussion to the ty -plane.

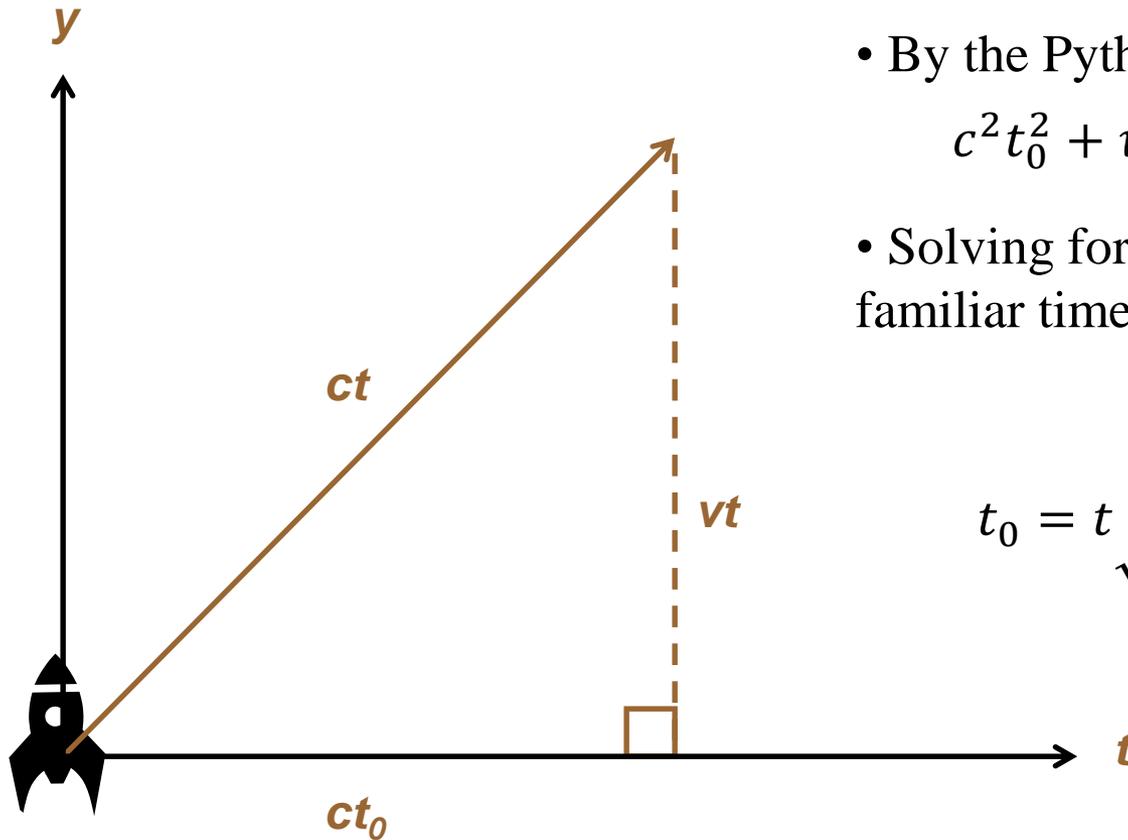
A Rocket "Sitting" on the Launch Pad



Takes Off with Spatial Velocity v ; Overall Velocity c



The Time Dilation Formula



- By the Pythagorean theorem

$$c^2 t_0^2 + v^2 t^2 = c^2 t^2$$

- Solving for t_0 , results in the familiar time dilation formula.

$$t_0 = t \sqrt{1 - \frac{v^2}{c^2}}$$

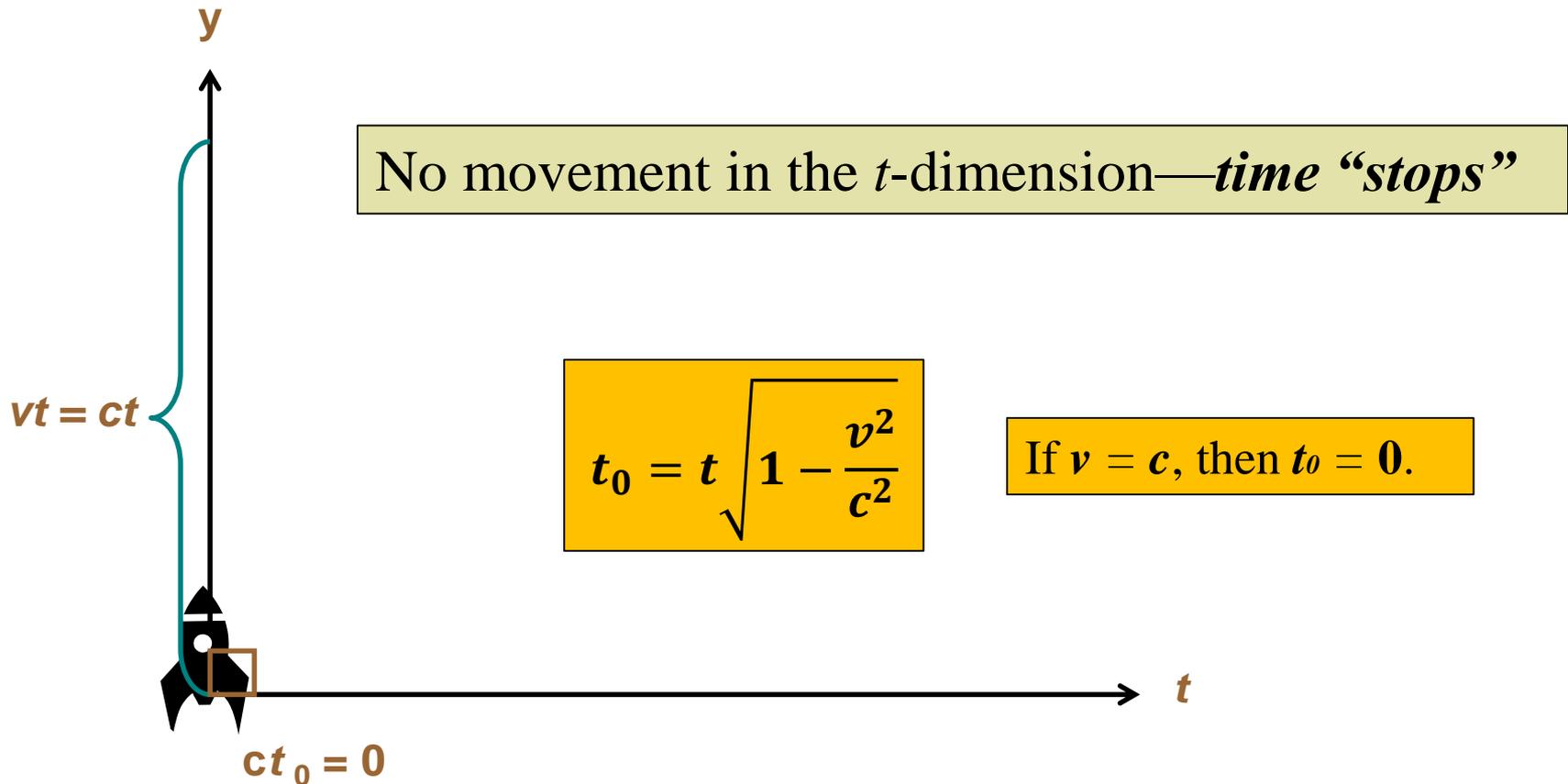
Non-Constant Passage of Time

- If the astronaut is moving at 80% of the speed of light in the y -direction, then

$$t_0 = t \sqrt{1 - \frac{v^2}{c^2}} = t \sqrt{1 - \left(\frac{0.8c}{c}\right)^2} = 0.6t .$$

If we experience $t = 10$ years, the astronaut experiences $t_0 = 6$ years.

Traveling at the Speed of Light c in a Spatial Dimension



Equations Before Observations

- “Mathematics can lead us in a direction we would not take if we only followed up physical ideas by themselves.”

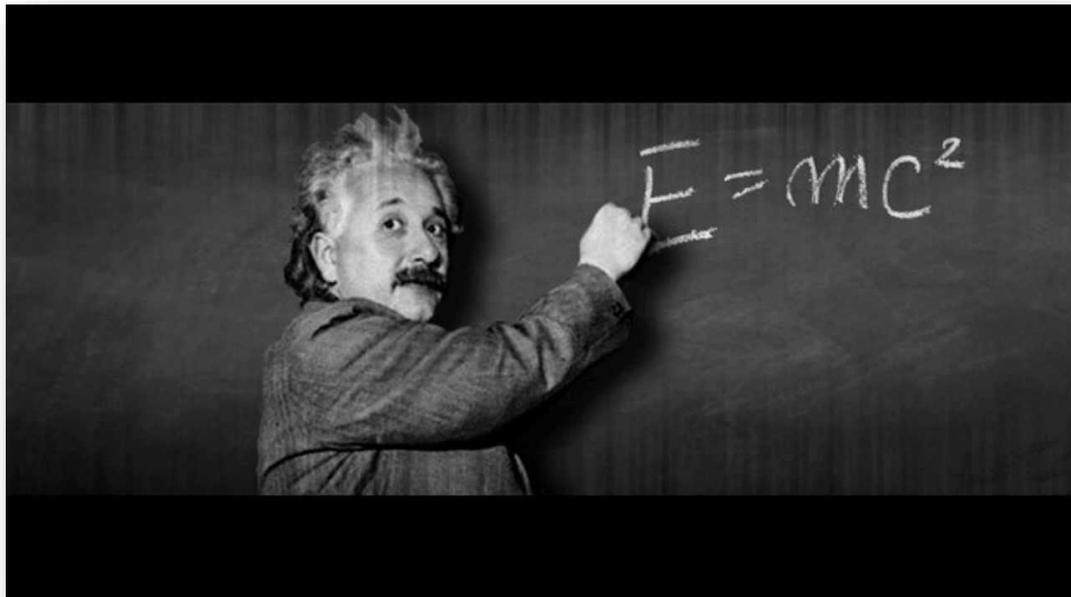


--Paul Dirac,

A founder of quantum theory

Mathematics is Amazing!

- Math resides in two worlds:
pure thought and *real-world phenomena*.
- Math uses symbols to derive more symbols, and these derived symbols can describe reality.



Stretching the Limits: String Theory



String Theory: Higher Dimensions

- Strings:
 - Smallest unit of matter (no points)
 - One-dimensional
 - Vibrate in 9 spatial dimensions and 1 time dimension

Why So Many Dimensions?

- *Modular functions*: invented 100 years ago by Ramanujan.
- Modular functions satisfy the same identities that strings must satisfy when changing their vibrational patterns.
- *Modular functions work only 10 or 26 dimensions.*





Why Do We Only “See” Three Spatial Dimensions?

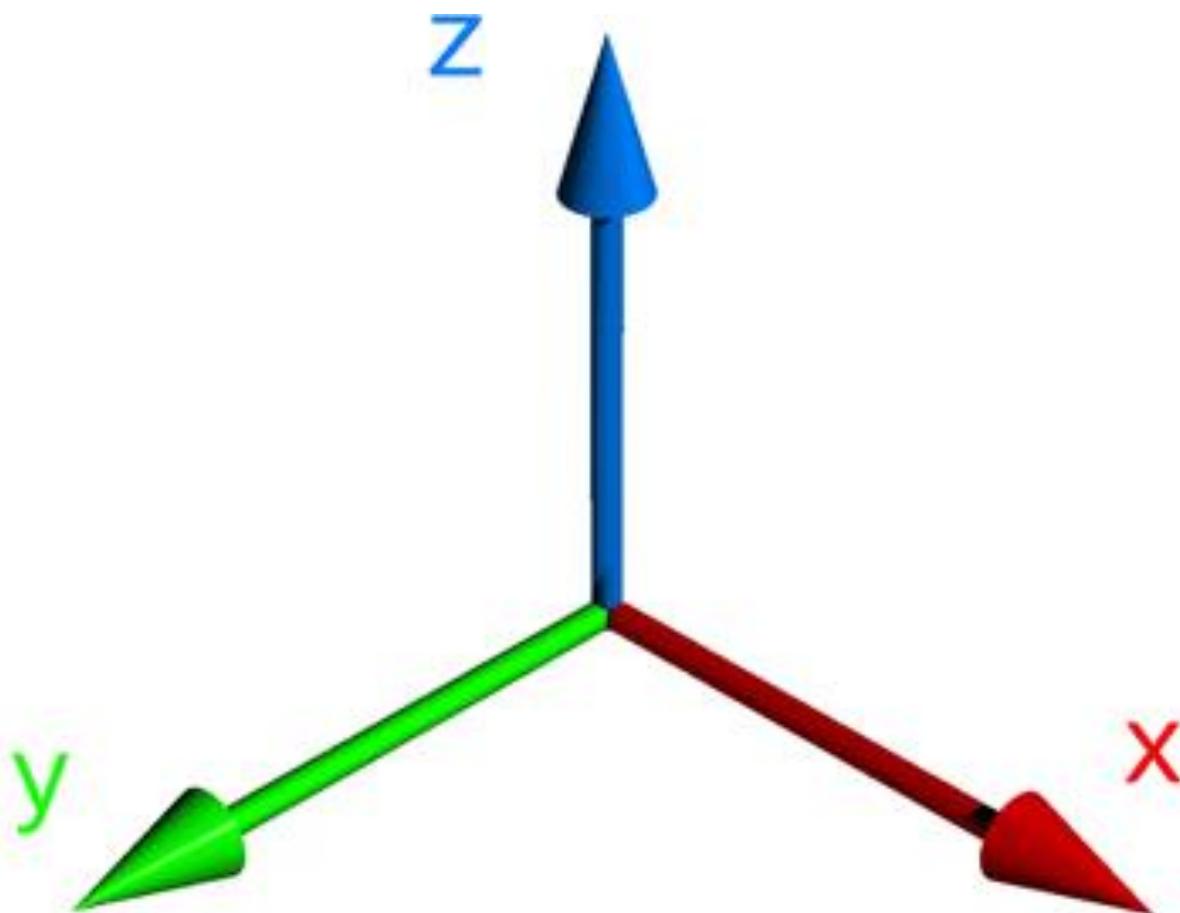
- There are 9 spatial and 1 time dimension. Only three of the spatial dimensions are “large.”

Curled Dimensions?

- Of the 9 spatial dimensions, six are small, or “curled.”

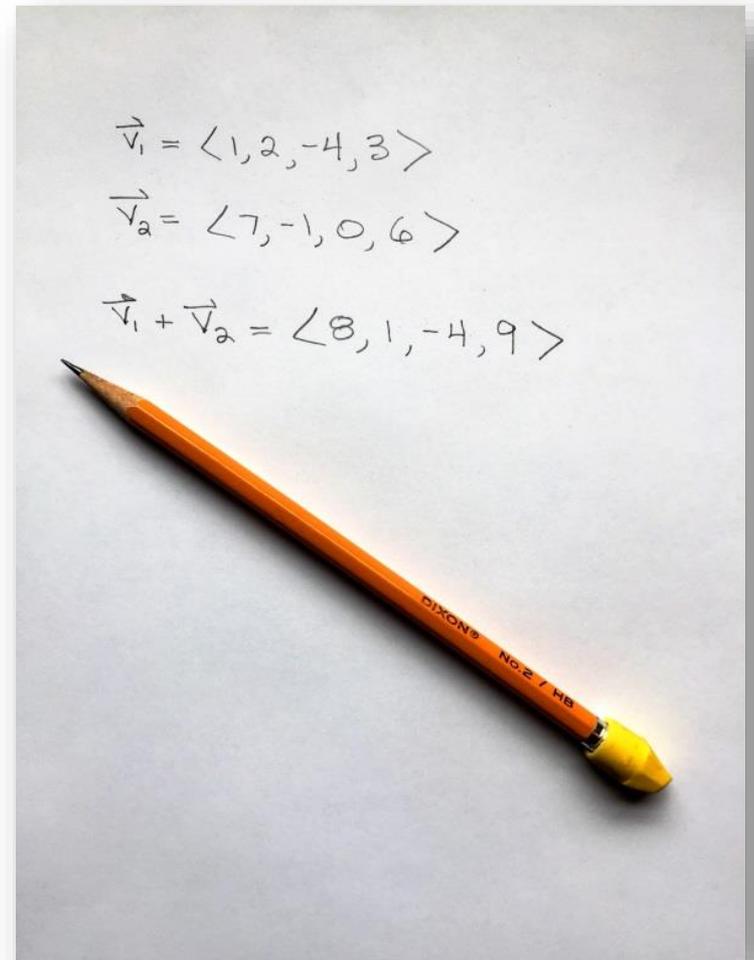
A bug could disappear in a curled dimension by walking around the cable.

Ten Dimensions?



Higher Dimensions

- Think about this:
- *“How is it that on a 2-dimensional sheet of paper, we can accurately describe n-dimensional space?”*



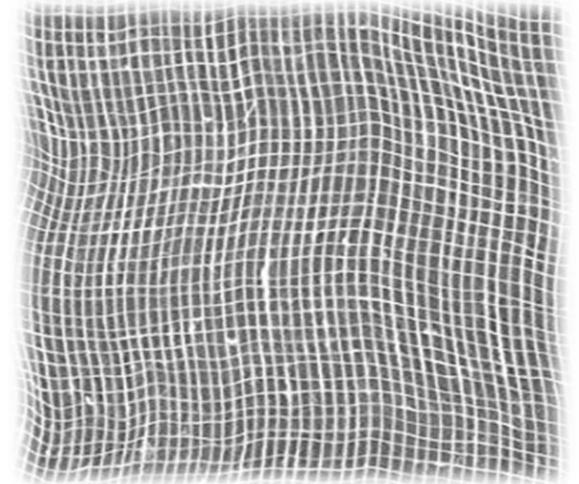


Discrete Space and Time

- Because strings have an extremely small, but finite size, points do not exist in space.
- *Both points and continuous time may be mathematical abstractions living outside physical reality.*

Discrete Space and Time?

- Space-time may be a “fabric.”
- What is the distance between the threads in the fabric?
 - The smallest unit of length: 10^{-33} cm.
 - The smallest unit of time: 10^{-43} sec.
- **“Like sands through the hourglass, so are the Days of Our Lives.”**





So Is Math Always Contained Within Reality?

- **Question:** If space is discrete, then are there other concepts, such as continuity, that “do not exist” in physical reality?

**Mathematics:
Invented or Discovered?**

An Age-Old Debate

- ❖ Some people believe that you cannot separate mathematics from the physical world, or human experience, so math is *invented*.
- ❖ Others believe that mathematics is *discovered*.
- ❖ Max Tegmark discusses this second belief in the following clip.
- ❖ https://www.youtube.com/watch?v=UKyth_yoJBc



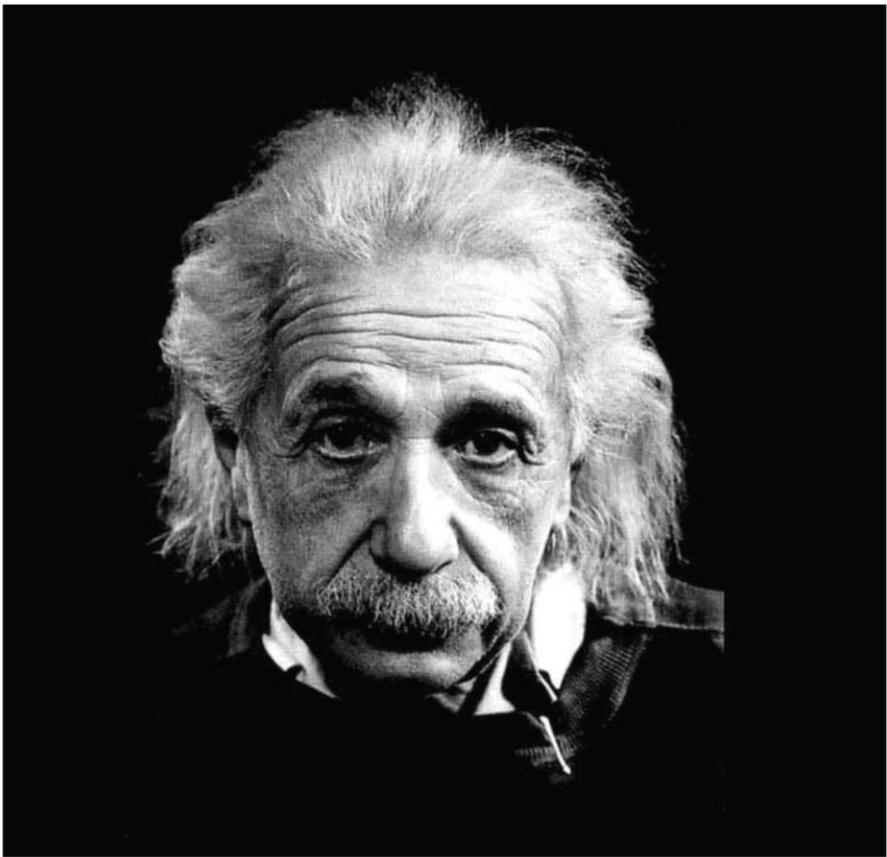
Max Tegmark's Theory

- **Math is Discovered:** Physical reality resides within mathematical structures.
- *Mathematics transcends physical reality.*

Is Reality a Mathematical Structure?

<https://www.youtube.com/watch?v=PTF-hHGbQ6s>

Einstein's Dream



“It is my conviction that pure mathematical construction enables us to discover the concepts and the laws connecting them, which gives us the key to understanding nature. In a certain sense, therefore, I hold it true that pure thought can grasp reality, as the ancients dreamed.”

—Albert Einstein

Thank you for attending!

See www.garyrockswold.net
for more information.